Diagrammar and metamorphosis of coset symmetries in dimensionally reduced type IIB supergravity

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Abstract

Studying the reduction of type IIB supergravity from ten to three space-time dimensions we describe the metamorphosis of Dynkin diagram for gravity line "caterpillar" into a type IIB supergravity "dragonfly" that is triggered by inclusion of scalars and antisymmetric tensor fields. The final diagram corresponds to type IIB string theory E_8 global symmetry group which is the subgroup of the conjectured E_{11} hidden symmetry group. Application of the results for getting the type IIA/IIB T-duality rules and for searching for type IIB vacua solutions is considered.

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One of the problems which has been actively studied last time is to identify the underlying Superstring/M-theory symmetry group [1, 2, 3]. Knowing the group is the essential step in passing from higher energies and dimensions to phenomenologically relevant vacua of M-theory and is the bypass for bringing to light the hidden but essential ingredients of higher dimensional theory that might help uncover its true non-perturbative structure. The evidence on possible group-theoretical ground of M-theory comes from studying the coset symmetries in dimensionally reduced D=11 supergravity as the M-theory low-energy limit. It was realized long ago [4] that the global symmetry groups of D=11 supergravity toroidally compactified down to four space-time dimensions fall into the class of exceptional groups E_n with $n \leq 7$. Subsequent reduction of D=11 supergravity to three [5] and to two [6] dimensional space-times (see also [7]) revealed E_8 and E_9 global symmetry structure. The role of E_{10} as the hidden symmetry group of D=11 supergravity compactified onto ten-dimensional torus was emphasized in [8] and this conjecture has been proved in [7].

The important step in relating the global symmetry groups of toroidally reduced D=11 supergravity to the true but hidden symmetry group of the theory in eleven dimensions was done in [9]. There was discovered the exceptional geometry of D=3 maximal supergravity that allowed to reformulate D=11 supergravity in a E_8 invariant way. Together with previously obtained results of [10] and [11] this observation gave the strong evidence in favor of searching for the exceptional geometry of M-theory based on a symmetry group which shall contain as a subgroup the E_n sequence of global low-dimensional symmetries with $n \leq 10$. Following previous experience it could be naively expected to have, after "compactifying" the time, a hidden symmetry group whose algebra is of the rank eleven and includes E_9 and E_{10} as subalgebras. Since E_{10} is the hyperbolic Kac-Moody algebra [12] which contains E_8 and its affine extension $E_9 \sim E_8^+$ as subalgebras and is called the over-extension of E_8, i.e. $E_{10} \sim E_8^{++}$, an M-theory hidden symmetry group should also be an extension of hyperbolic Kac-Moody algebras. Such a generalization studied in [2], [13], [14], [15] has been christened the "very-extension" of E_8 or $E_{11} \sim E_8^{+++}$ (see also [16], [17]).

The relevance of E_{11} to the non-linear realization of supergravities has been demonstrated for the bosonic subsectors of higher dimensional maximal supergravities [1], [18]. A curious result is that the global hidden symmetry group of type IIB theory turns out to be the same as for M&A theories related to each other by dimensional reduction. Since the type IIB theory is not related to M-theory via straightforward dimensional reduction on a circle the validity of E_{11} as the type IIB theory global symmetry group can be established in studying the coset structure of dimensionally reduced type IIB supergravity. Substantial fragments of this structure has been found in [19], [20], [21], [14], [22].

The aim of the present paper is to re-collect the fragments in a systematic way during the application of "anti-oxidation" strategy of [23] and to emphasize the points that were omitted from previous considerations. We are getting started with ten-dimensional theory compactifying it to three space-time dimensions. The special role of three-dimensional space-time in establishing the hidden symmetries of higher-dimensional supergravities has been emphasizing for a long time since there is no gravity degrees of freedom in D=3 and dualising gauge fields we get the theory whose dynamics is completely determined by scalar degrees of freedom. Depending on the original higher-dimensional sub-sectors of fields the scalars parameterize different G/H coset spaces. Identifying the global group G for different subsectors of type IIB supergravity is our main task.

Let us begin our quest of the hidden symmetry group of type IIB superstring theory with

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the following action describing dynamics of the bosonic subsector of fields entering the type IIB supergravity multiplet

$$S = \int_{\mathcal{M}^{10}} [R * \mathbf{1} + \frac{1}{2} d\phi_0 * d\phi_0 + \frac{1}{2} e^{2\phi_0} d\chi_0 * d\chi_0$$

$$-\frac{1}{2} e^{-\phi_0} H^{(3)} * H^{(3)} - \frac{1}{2} e^{\phi_0} \tilde{H}^{(3)} * \tilde{H}^{(3)} + \frac{1}{4} \tilde{H}^{(5)} * \tilde{H}^{(5)}$$

$$+ \frac{1}{2} A^{(4)} dB^{(2)} dA^{(2)} + \mathcal{L}_{PST}]. \tag{1}$$

The first term of (1) corresponds to Einstein-Hilbert term, $\int_{\mathcal{M}^{10}} *\mathbf{1} \equiv \int d^{10}x\sqrt{-g}$, ϕ_0 and χ_0 are the dilaton and axion scalars. $H^{(3)}$ and $\tilde{H}^{(3)}$ are the field strengths of NS and RR gauge fields $B^{(2)}$ and $A^{(2)}$

$$H^{(3)} = dB^{(2)}, \qquad \tilde{H}^{(3)} = dA^{(2)} - H^{(3)}\chi_0. \tag{2}$$

As well as the scalars they cast the doublet under $SL(2,R)$ global symmetry group of type IIB theory. $\tilde{H}^{(5)}$ is the self-dual field strength of the $SL(2,R)$ singlet RR field $A^{(4)}$

$$\tilde{H}^{(5)} = dA^{(4)} - H^{(3)}A^{(2)}, \qquad \tilde{H}^{(5)} = *\tilde{H}^{(5)}. \tag{3}$$

The last term which is not important for the discussion in what follows encodes the self-duality condition [24, 25]. Since we have made the choice of differential form notation the wedge product between forms has to be assumed.

The first step in completing our task is to recover the following structure of action

$$S = \int_{\mathcal{M}^3} \left[\frac{1}{2} d\vec{\phi} * d\vec{\phi} + \frac{1}{2} \sum_{\vec{\alpha}} e^{\vec{\alpha}\cdot\vec{\phi}} d\chi_{\vec{\alpha}} * d\chi_{\vec{\alpha}} \right] + \dots, \tag{4}$$

which is obtained from (1) after performing dimensional reduction on T^7. Here $\vec{\phi} = (\phi_0, \phi_1, \dots, \phi_7)$ is the dilaton vector comprised of the original dilaton ϕ_0 and those appeared during dimensional reduction. The $\vec{\alpha}$ are constant eight-vectors which label additional (axionic) scalar fields $\chi_{\vec{\alpha}}$. The difference between two types of scalars consists in sources of their appearance due to dimensional reduction and different types of interactions they possess [26]. The axions come from non-diagonal part of the Kaluza-Klein metric and from dualizing the higher rank tensor fields. They possess only derivative interactions. On the contrary the scalars associated with dilatons comes from diagonal part of the metric and can have non-derivative interactions as in (4). The form of the action (4) is a signal that the scalars parameterize a G/H coset space if of course one can identify the axion counting vectors as positive roots of a group G. The global symmetry group G is uniquely defined by the Cartan matrix constructed out of the simple roots [26].

To reach the action (4) we will use the same strategy as in [23]. To this end one has to take into account the standard rules of step by step toroidal reduction [27]

$$\int d^D x \, eR \longrightarrow \int d^{D-1}x \, eR + \int_{\mathcal{M}^{D-1}} [\frac{1}{2} d\phi_1 * d\phi_1$$

$$+ \frac{1}{2} e^{-2(D-2)\alpha_{D-1}\phi_1} F_1^{(2)} * F_1^{(2)}] \tag{5}$$

with

$$\sqrt{2(D-2)(D-3)} \cdot \alpha_{D-1} = 1, \tag{6}$$

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and

$$\int_{\mathcal{M}^D} \frac{1}{2} F^{(n)} * F^{(n)} \longrightarrow$$

$$\int_{\mathcal{M}^{D-1}} [\frac{1}{2} e^{-2(n-1)\alpha_{D-1}\phi_1} F_1^{(n)} * F_1^{(n)} + \frac{1}{2} e^{2(D-1-n)\alpha_{D-1}\phi_1} F_1^{(n-1)} * F_1^{(n-1)}]. \tag{7}$$

The effect of transgression consisting in appearing new terms in the reduced field strength $F_1^{(n)} = dA_1^{(n-1)} + \dots$ and of having the Chern-Simons term in type IIB supergravity action are denoted by the ellipsis in (4) and may be safely ignored [23] since they have not an influence on the results in what follows.

Let us get started with pure gravity case. Performing the reduction in step by step manner one can recover six seven-dimensional simple root vectors $\vec{\alpha}_k$ having the following structure [23]

$$\vec{\alpha}_k = (0, \dots, 0, -2(8-k)\alpha_{9-k}, 2(6-k)\alpha_{8-k}, 0, \dots, 0) \tag{8}$$

with α's from (6), $k = 0, \dots, 5$ zeros on the left and $(5-k)$ zeros on the right. It is easy to verify that

$$\vec{\alpha}_i \cdot \vec{\alpha}_k = \begin{cases} 4, & i = k \\ -2, & |i-k| = 1 \\ 0, & |i-k| \geq 2 \end{cases} \tag{9}$$

All other roots coming from this subsector of type IIB supergravity are not simple and can be expressed as a linear combination of simple roots with non-negative coefficients.

One more simple root vector comes from dualising the Kaluza-Klein vector field which appeared in the first step of reduction from ten to three and having the following form

$$\vec{\delta} = (16 \cdot \alpha_9, 2 \cdot \alpha_8, 2 \cdot \alpha_7, \dots, 2 \cdot \alpha_3). \tag{10}$$

One can check that

$$\vec{\delta} \cdot \vec{\alpha}_k = \begin{cases} 0, & k \neq 0 \\ -2, & k = 0, \end{cases} \qquad \vec{\delta} \cdot \vec{\delta} = 4, \tag{11}$$

and other roots that come from dualising the rest of the Kaluza-Klein vectors are not simple. Denoting $\vec{\delta}$ as $\vec{\alpha}_{(0)}$ one can construct the Cartan matrix

$$A_{ij} = 2 \frac{\vec{\alpha}_i \cdot \vec{\alpha}_j}{\vec{\alpha}_i \cdot \vec{\alpha}_i}, \qquad i = (0), 0, \dots, 5 \tag{12}$$

that corresponds to the A_7 Dynkin diagram. This diagram is that of $SL(8)$ group and is called the gravity line.
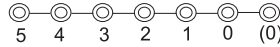

Figure 1: Gravity line "caterpillar" of A_7.

Let us extend our analysis to include the dilaton-axion sector of type IIB supergravity. Since we have ϕ_0 from the beginning we shall extend our simple root vectors (8), (10) with one additional column with zero on the left, i.e.

$$\vec{\alpha}_{(0)} = (0, 16\alpha_9, 2\alpha_8, 2\alpha_7, \dots, 2\alpha_3),$$

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etc. Hence we should deal with eight-dimensional simple root vectors. And since we have axion χ_0 from the beginning we have also one additional eight-dimensional root vector

$$\vec{\epsilon} = (2, 0, \ldots, 0). \tag{13}$$

Clearly,

$$\vec{\epsilon} \cdot \vec{\epsilon} = 4, \qquad \vec{\epsilon} \cdot \vec{\alpha}_k = 0, \ \forall k. \tag{14}$$

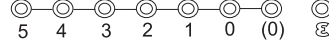The Cartan matrix extended by new root corresponds to the following Dynkin diagram

Figure 2: Beginning of the "caterpillar's" metamorphosis.

that encodes the $SL(8) \otimes SL(2, R)$ group structure of this sector of fields of type IIB supergravity. In the language of Dynkin diagrams the first node on the right corresponds to the $SL(2, R)$ group.

The next step is to extend our system of roots with inclusion of $H^{(3)}$ and $\tilde{H}^{(3)}$ tensor fields. One more simple root is coming from the reduction of the NS field kinetic term [23]

$$\vec{\beta} = (-1, 12 \cdot \alpha_9, 12 \cdot \alpha_8, 0, \ldots, 0). \tag{15}$$

It is easy to check that

$$\vec{\beta} \cdot \vec{\alpha}_k = \begin{cases} 0, & k \neq 1 \\ -2, & k = 1 \\ +2, & k = (0), \end{cases} \qquad \vec{\beta} \cdot \vec{\beta} = 4, \quad \vec{\beta} \cdot \vec{\epsilon} = -2. \tag{16}$$

As such the root $\vec{\alpha}_{(0)}$ is not simple anymore since the off-diagonal entries of the Cartan matrix are negative integers or zero. It is merely a technical point to establish the absence of other simple roots which could possibly come from the dualisation of the NS 2-form gauge field and the rest of the fields of type IIB multiplet and their dualisation. Therefore at this stage of our study we arrive at the following diagram.
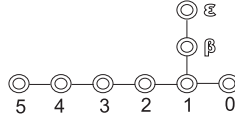

Figure 3: End of metamorphosis. "Dragonfly" is fully fledged.

This diagram is topologically equivalent to the E_8 Dynkin diagram and therefore the latter is the global symmetry group of type IIB theory compactified to three space-time dimensions.
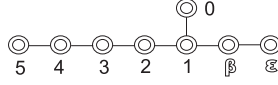
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Figure 4: Type IIB E_8-like diagram.

Let us turn now to the applying the results obtained so far. We will describe first the interpretation of T-duality rules in the language of Dynkin diagrams (see also [21], [14]). Doing the calculations outlined above one arrives at the following diagram that encodes the coset structure of type IIA supergravity reduced to three space-time dimensions. Here we have chosen a slightly different notation to indicate the tensor fields from which the simple roots came.
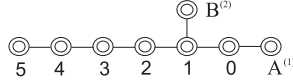

Figure 5: Type IIA E_8 diagram.

Comparing Fig. 5 to Fig. 4 one can observe that two diagrams coincide along the gravity line from the nodes 1 to 5. This is just an indication of having the same gravity sub-sector for two theories in D=9 space-time dimension. To have the same theories in D=9 we have to identify the node corresponding to type IIA NS 2-form $B^{(2)}$ with that of type IIB gravity node, the type IIA gravity node with the node of type IIB NS 2-form field and the one of type IIA Kaluza-Klein vector field with that of type IIB axion. This identification corresponds to seminal T-duality rules (cf. e.g. [31])

$$i_z B^{(2)}_{IIA} \cong (i_Z g)_{IIB}, \quad (i_z g)_{IIA} \cong i_Z B^{(2)}_{IIB}, \quad i_z A^{(1)} \cong \chi_0. \tag{17}$$

Another important point in playing with Dynkin diagrams is the possibility of identifying the relevant $AdS \times S$ vacuum configurations [22]. Couple of years ago it was a breakthrough in constructing the consistent non-linear Kaluza-Klein ansätze for spherical dimensional reduction of supergravities (see [26] and [28] for review). A systematic group-theoretical ground indicating the possibility of such reductions is still lacking though it was formulated the criterion of consistency of the reduction on S^n based on the possibility to enhance the global symmetry group after T^n reduction due to "conspiracy" of scalars.

Essential step in searching for the non-linear Kaluza-Klein ansätze for spherical reductions is figuring out the possibility of having the $AdS \times S$ vacuum configuration. Recently it was proposed the method of examining such a possibility based on considering the appropriate Kac-Moody algebras [22]. In the context of type IIB supergravity the evidence of $AdS_5 \times S^5$ vacuum configuration is based on manipulations with E_7 diagram
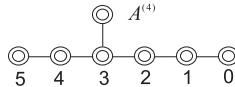

Figure 6: Type IIB E_7 diagram.

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extended with three additional nodes on the left, i.e. with E_7^{+++} diagram (cf. Fig. 7).
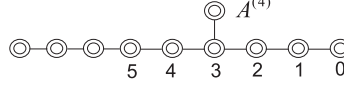

Figure 7: E_7^{+++} diagram.

The origin of the E_7 is easy to explain since the latter corresponds to a "larva" in the metamorphosis of Fig. 1 to Fig. 3. Such a diagram comes from the subsector of fields consisting of gravity and self-dual 4-form gauge field. It is worth mentioning that the root $\vec{\xi}$ coming from the reduction of $\tilde{H}^{(5)}$ field strength and the one coming from its dualisation obey the following relations

$$\vec{\xi} \cdot \vec{\alpha}_k = \begin{cases} 0, & k \neq 3 \\ -2, & k = 3 \\ +2, & k = (0), \end{cases} \qquad \vec{\tilde{\xi}} \cdot \vec{\alpha}_k = \begin{cases} 0, & k \neq 3 \\ -2, & k = 3 \\ +2, & k = (0), \end{cases}$$

$$\vec{\xi} \cdot \vec{\xi} = 4, \qquad \vec{\tilde{\xi}} \cdot \vec{\tilde{\xi}} = 4, \qquad \vec{\xi} \cdot \vec{\tilde{\xi}} = 4. \tag{18}$$

Hence it is a matter of taste which one is selected to be the simple root that is the remnant of the $\tilde{H}^{(5)}$ self-duality. As soon as the choice was made another root is no longer simple.

Skipping the details of manipulations with E_7^{+++} diagram that leads to the $AdS_5 \times S^5$ configuration (we refer the reader to the original paper [22]), it is worth mentioning that the subset of fields leading to the E_7^{+++} is precisely the one for which the existence of the non-linear ansatz for the S^5 dimensional reduction [29] was proved! Another example of having the non-linear ansätze for $AdS_{D-3} \times S^3$ and $AdS_3 \times S^{D-3}$ configurations was established for the bosonic string theory [30] which includes graviton, dilaton and 2-rank gauge field in the massless sector. This subsector enters the type IIB supergravity and corresponds to the D_8^{+++} Kac-Moody group. One can verify following the approach of [22] that such vacuum solutions are indeed the case.

To summarize, we have traced the metamorphosis of Dynkin diagrams representing the symmetries of different sub-sectors of dimensionally reduced type IIB supergravity. This provides the link to the results obtained in the framework of non-linear realization of type IIB supergravity [18] and of searching for the M-theory hidden symmetry group [1, 2, 3, 14, 15] as well as to the results obtained by use of the oxidising technique [20, 21]. The graphical representation of the coset symmetries in dimensionally reduced supergravities encodes a lot of information on the matter field content of a theory, the relevant low-dimensional vacua and dualities between different supergravities and is therefore the very useful tool in studying the hidden symmetry structure of Superstring/M-theory.

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